IB Math

Summer

Packet

Due: 1st day of school

Summer 2018

Dear Incoming Student,

Congratulations on accepting the challenge of taking International Baccalaureate Mathematics Standard Level (IB Math SL). I have prepared this packet to give you additional information about the course and to help you prepare to be successful in IB Math SL.

IB Math SL is a rigorous course that covers topics in Algebra, Functions, Trigonometry, Vectors, Statistics and Probability, and Calculus. In May, you will take an IB Math SL exam certifying that you have mastered the content of a college-level curriculum. Depending on where you go to school, you may be granted college credit based on the score you receive on the exam. You will also be writing an Internal Assessment. The Internal Assessment is worth 20% of your IB Exam grade.

IB Math SL is a demanding course. You should be proud of the work you have done to prepare yourself thus far. My goal is to help you grow as a mathematician as much as possible over the next year, but I can only be a guide. It is up to you to put forth the consistent effort necessary to succeed in this course. Note: If you struggled in Algebra 2 Honors, this course will be extremely difficult for you. If you have trouble with this packet you might want to see your counselor about a schedule change.

I have developed a summer assignment to help you refresh some skills necessary for success in this course. Please complete the entire packet, showing all of your work carefully, **without use of a calculator wherever possible**. Many of our exams this year will have a non-calculator component in order to prepare for that aspect of the exam in May. Everything in this packet should be review for you. If you need help with any of the topics, check out online resources like Khan Academy or You Tube. Be sure to use the attached formula sheet as well. Whenever giving approximate answers, use 3 significant figures. IB uses significant figures not a specified number of decimal places. I also included a resource sheet with some notation and command terms that you should start becoming familiar with.

Your summer assignment is due on the first day of class. You must show all work to receive credit. Please make sure that you have mastered the material in this packet to ensure your success. All the material in this packet is content that will be on the IB Exam in May as well as on exams periodically throughout the year. Because we only have a year and have new topics to learn, we won't go over this material much more throughout the year. **You will be given a test on this material the THIRD DAY of school!!!**

I look forward to getting to know you and working with you next year. Please feel free to email me at any time with questions. Have a relaxing, enjoyable summer. I look forward to studying mathematics with you next year!

Sincerely,

Mrs. Kimberly Steere
ksteere@theproutschool.org

Equations given in Class and on the IB Exam

Topic 1—Algebra

1.1	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
	The n^{th} term of a geometric sequence	$u_n = u_1 r^{n-1}$
	The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$, $r \neq 1$
	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r}$, $\lvert r \rvert < 1$
1.2	Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b$
	Laws of logarithms	$\log_c a + \log_c b = \log_c ab$
		$\log_c a - \log_c b = \log_c \frac{a}{b}$
		$\log_c a^r = r \log_c a$
	Change of base	$\log_b a = \frac{\log_c a}{\log_c b}$

Topic 2—Functions and equations

2.4	Axis of symmetry of graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry $x = -\frac{b}{2a}$
2.6	Relationships between logarithmic and exponential functions	$a^x = e^{x \ln a}$ $\log_a a^x = x = a^{\log_a x}$
2.7	Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$
	Discriminant	$\Delta = b^2 - 4ac$

Interval Notation (Not given in class or on the exam... just a refresher)

Description	Interval notation	Description	Interval notation	Description	Interval notation
$x > a$	(a, ∞)	$x \leq a$	$(-\infty, a]$	$a \leq x < b$	$[a, b)$
$x \geq a$	$[a, \infty)$	$a < x < b$	(a, b) - open interval	$a < x \leq b$	$(a, b]$
$x < a$	$(-\infty, a)$	$a \leq x \leq b$	$[a, b]$ - closed interval	All real numbers	$(-\infty, \infty)$

MEMORIZE NOTATION/COMMAND TERMS LIST

NOTATION

Number Sets	$\mathbb{N}$	the set of positive integers and zero, $\{0,1,2,3,\ldots\}$
	$\mathbb{Z}$	the set of integers, $\{0,\pm1,\pm2,\pm3,\ldots\}$
	$\mathbb{Z}^+$	the set of positive integers, $\{1,2,3,\ldots\}$
	$\mathbb{Q}$	the set of rational numbers (any # that can be written as a fraction)
	$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x>0\}$
	$\mathbb{R}$	the set of real numbers
	$\mathbb{R}^+$	the set of positive real numbers, $\{x \mid x \in \mathbb{R}, x>0\}$
Absolute Value	IB will refer to this as modulus	
Line Segments	Line segment, $\overline{AB}$, will be written as $[AB]$	
Angles	We write Angle A as $\angle A$. IB will use the following notation: $\hat{A}$	
Repeating Decimals		

		IB Notation
$\frac{1}{3}$	$0.\overline{3}$	$0.\dot{3}$
.123123123123	$0.\overline{123}$	$0.\dot{1}2\dot{3}$

Slope	IB will refer to this as the gradient

COMMAND TERMS

Calculate	Obtain a numerical answer showing the relevant stages in the working
Determine	Obtain the only possible answer
Draw	Represent by means of a labeled, accurate diagram or graph, using a pencil. A ruler should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted and joined in a straight line or curve.
Find	Obtain an answer, showing relevant stages in that working.
Hence	Use the proceeding work to obtain the required result
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.
Show that	Obtain the required result (possible using information given) without the formalily of proof. "Show that" questions do not generally require the use of a calculator.
Sketch	Represent by means of a diagram or graph (labelled as appropriate) The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Solve	Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.
Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.

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QUADRATICS

Section A: Factor each quadratic. If the quadratic cannot be factored, write "prime."

1. $x^2 - x - 2$
2. $x^2 + 3x - 4$
3. $8x^2 - 50y^2$
4. $3x^2 - 5x + 2$
5. $2x^2 - x - 6$
6. $x^3 - 3x^2 - 18x$

Section B: Solve each equation using any method except graphing or guess and check.

1. $x^2 + 25 = 10x$
2. $x^2 + 3x - 1 = 0$
3. $x + \frac{12}{x} = 7$
4. $x^2 + 2 = 9$
5. $x^2 - 5x = 0$
6. $36x^2 - 25 = 0$

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Section C: State the following for each of the given equations: axis of symmetry, vertex, direction of opening, x-intercepts, and y-intercepts. Then sketch the graph using that information.

1. $y = -2(x+2)(x-1)$
2. $y = 0.5(x-2)^2 - 4$
3. $y = 2x^2 + 6x - 3$

Section D: Find the values of p such that the equations below have the given characteristics. Hint: Use the discriminant.

1. Two different real roots
   $px^2 + 5x + 2 = 0$
2. Two equal real roots
   $2x^2 - 3x + p = 0$
3. No real roots
   $px^2 - 4px + 5 - p = 0$

Section E: Use your graphing calculator to find the following

1. Solve $3x^2 - x - 5 = 0$
2. Intersection points of $y = -x^2 - 5x + 3$ and $y = x^2 + 3x + 11$

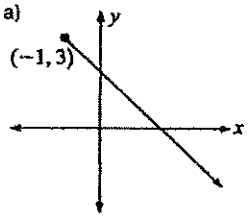
IB Math Standard Level Summer Assignment

FUNCTIONS

Section A: For each of the following find the domain and range without using a calculator.

1.

a)



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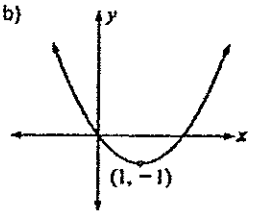

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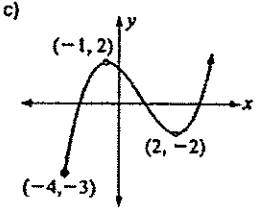

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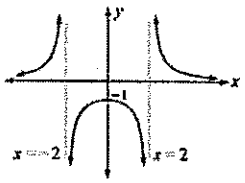

2.

a) $f(x) = \sqrt{x}$ b) $f(x) = \sqrt{4 - x}$ c) $y = 5x - 3x^2$ d) $y = \frac{x+4}{x-2}$

Section B: Find the inverse of each function.

1. $f(x) = 2x + 1$

2. $f(x) = \frac{x^2}{3}$

3. $g(x) = \frac{5}{x-2}$

4. $g(x) = \sqrt{4 - x} + 1$

5. If the point (2, 7) is on the graph of $f(x)$, what point must be on the graph of $f^{-1}(x)$?

6. Explain, in complete sentences, the relationship between a function and its inverse.

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Section C: Let $f(x) = 2x^2 - 1$; $g(x) = 3x$ and $h(x) = 5 - x$. Find the following.

1. $f(-3)$
2. $(f \circ g)(x)$
3. $(h \circ f)(x)$
4. $(f \circ h)(x + 1)$
5. $(g \circ h)(4)$
6. $(f \circ f)(-1)$

Section D: Answer the following questions concerning equations of lines

1. What is the slope, x-intercept, and y-intercept of the equation $5x - 4y = 8$?

2. What is the slope-intercept form of the equation of the line between the points (4, 3) and (7, –2) ?

3. What is the slope-intercept form of a line perpendicular to $y = -2x + 9$ passing through the (4, 7) ?

Section E: Find the horizontal & vertical asymptotes and holes (if applicable) of the following.

1. $y = \frac{1}{2x-5}$
2. $y = \frac{x^2-5}{2x^2-12}$
3. $y = \frac{x^2+2x-3}{x^3+6x^2-7x}$

Section F: For each pair of functions $f(x)$ and $g(x)$, describe the transformations that would transform $f(x)$ into $g(x)$.

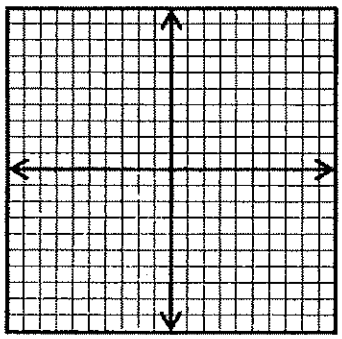
1. $f(x) = x^2$;
   $g(x) = (x-5)^2 + 2$

2. $f(x) = \sqrt{x}$;
   $g(x) = \sqrt{3x} - 10$

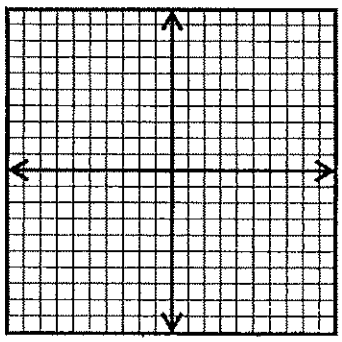
3. $f(x) = e^x$;
   $g(x) = -5(e)^{x-1}$

Section G: Graph each function, clearly showing its key features (maxima, minima, and intercepts). Identify its domain and range. (Remember: No calculator!)

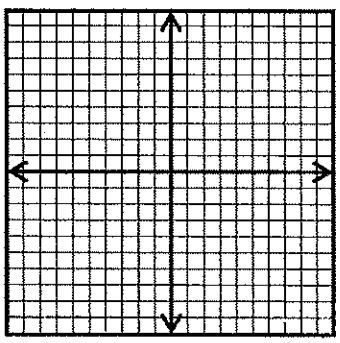
1. $f(x) = x^2 - 5$



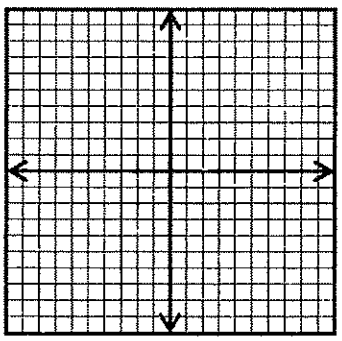
2. $f(x) = 3x - 4$



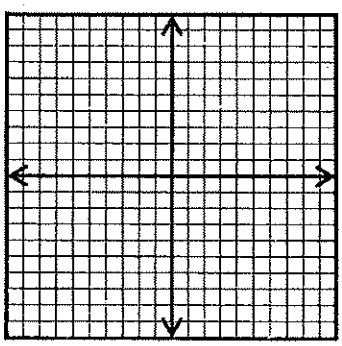
3. $f(x) = x^3 + 1$



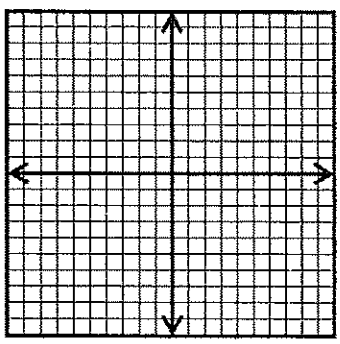
4. $f(x) = \sqrt{x+6}$



5. $f(x) = |x-1| + 3$



6. $f(x) = 2^x - 4$



IB Math Standard Level Summer Assignment

ALGEBRA

Section A: Simplify the following without a calculator.

1. $(2x^5)^{-3}$
2. $8^{2/3}$
3. $81^{-3/4}$
4. $\sqrt[3]{16x^3}$
5. $\sqrt{10x^2} \cdot \sqrt{70x^6}$
6. $\frac{\sqrt{72x^4}}{\sqrt{3x}}$
7. $\frac{5}{7-\sqrt{5}}$
8. $\sqrt{5} - 5\sqrt{125} - 7\sqrt{180}$

Section B: Solve using algebra.

1. $\begin{aligned} 3x + 7y &= 36 \\ x &= 5y - 10 \end{aligned}$
2. $\begin{aligned} 6x + 10y &= 32 \\ 4x - 2y &= 4 \end{aligned}$
3. $\begin{aligned} x &= y^2 \\ x - y &= 6 \end{aligned}$
4. $\begin{aligned} x^2 + y^2 &= 25 \\ y &= x^2 - 13 \end{aligned}$

Section C: Solve for x. Eliminate any extraneous solutions, if any.

1. $\sqrt{37 - 3x} = x - 3$
2. $-3(2x + 1)^3 = -192$
3. $\frac{x}{3} - \frac{5}{2} = \frac{-3}{x}$
4. $\frac{4x-1}{x+1} = x - 1$
5. $2|3x - 1| + 5 = -2x + 8$
6. $5(x - 3) \leq 8(x + 5)$

IB Math Standard Level Summer Assignment

EXPONENTIAL AND LOGARITHMIC EQUATIONS

Section A: Find the following without using a calculator.

1.

a) $\log_4 64$ b) $\log_2 1/4$ c) $\log_8 1$ d) $\log_9 3$ e) $\log_m m^6$ f) $\ln(e^{2x})$

Section B: Solve each equation for x or y.

1. $7 = 5^x$
2. $25e^{x/2} = 750$
3. $\log_2 y = 3$
4. $3\ln x + 2 = 0$
5. $\log_2 y + \log_2(y+1) = 1$
6. $4^y = 32$ (Solve without a calculator)

Section C: Answer the following questions about the equation $W = 2500\left(3^{-t/3000}\right)$ where W is the weight in gram of a radioactive substance after t years.

1. a) Find the initial weight
   b) Find the weight after 1500 years
2. Find how many years it takes to reduce its value 30%

IB Math Standard Level Summer Assignment '20

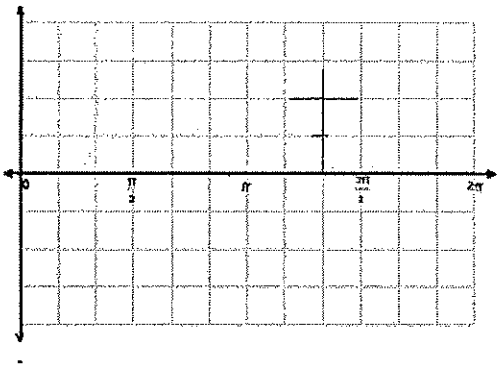
TRIGONOMETRIC FUNCTIONS

Section A: Find the exact value of each. Remember: No calculator!

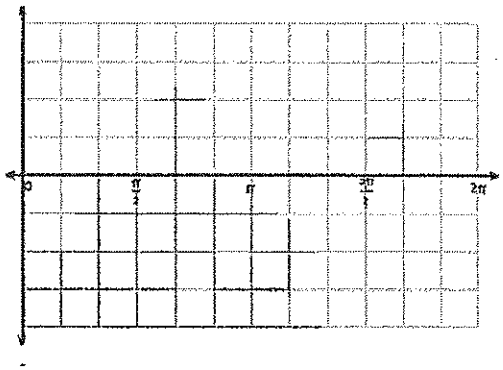
1. $\sin 60°$
2. $\tan 90°$
3. $\sin \pi$
4. $\tan\left(\frac{\pi}{3}\right)$
5. $\cos\left(\frac{7\pi}{6}\right)$
6. $\cos(-45°)$
7. $\tan 135°$
8. $\cos 300°$
9. $\sin\left(\frac{4\pi}{3}\right)$

Section B: Graph the functions below on the domain $0 \leq x \leq 2\pi$ (Remember: No calculator!)

1. $f(x) = \sin x$



2. $f(x) = \cos x$



Section C: Solve each trigonometric equation for $0 \leq x \leq 2\pi$.

1. $\sin x = -\frac{1}{2}$
2. $2\cos x = \sqrt{3}$
3. $4\sin^2 x = 3$

*Recall: $\sin^2 x = (\sin x)^2$

4. $\tan x = 1$