Math Summer Packet 2017

General instructions for all math Summer Packets:

- Unless the directions tell you otherwise, do all work on a separate sheet of paper in numerical order

- Use a pencil and show all work

- Due date: **Be prepared to hand in the Summer Packet on Tuesday, August 29th.**
  It will be counted as the first homework assignment of the year: Full credit if handed in and done well, no credit if not handed in, and at the teacher's discretion, half credit if incomplete or done badly.

- During the first week, summer packets will be returned with an answer sheet, and time permitted for questions

- **Assessment: Thursday or Friday, At the teacher's discretion**

- Not only are you expected to do this packet, you are expected to do it **well.** Use textbooks, online tutoring, etc. to help. Excuses such as "I forgot" or "I never had this" will not be accepted. If you want to be an Honors student, act like an Honors student.

Questions can be directed to Mr. Poulin: npoulin@theproutschool.org

IB Summer Packet

Summary of Chapter 1 Functions

Introducing functions

- A **relation** is a set of ordered pairs.
- The **domain** is the set of all the first numbers (x-values) of the ordered pairs.
- The **range** is the set of the second numbers (y-values) in each pair.
- A **function** is a relation where every x-value is related to a unique y-value.
- A relation is a function if any vertical line drawn will not intersect the graph more than once. This is called the **vertical line test**.

The domain and range of a relation on a Cartesian plane

Interval notation:
Use round brackets (,) if the value is not included in the graph or when the graph is undefined at that point (a hole or **asymptote**, or a jump).
Use square brackets [,] if the value is included in the graph.

- Set notation:

$\{x : x < 6\}$

The set of | x-values | such that | x is less than 6

Function notation

- $f(x)$ is read as 'f of x' and means 'the value of function f at x'.

Composite functions

- The composition of the function f with the function g is written as $f(g(x))$, which is read as 'f of g of x', or $(f \circ g)(x)$, which is read as 'f composed with g of x'.
- A **composite function** applies one function to the result of another and is defined by $(f \circ g)(x) = f(g(x))$.

Inverse functions

- The **inverse** of a function $f(x)$ is $f^{-1}(x)$. It reverses the action of the function.
- Functions $f(x)$ and $g(x)$ are inverses of one another if:
  $(f \circ g)(x) = x$ for all of the x-values in the domain of g and
  $(g \circ f)(x) = x$ for all of the x-values in the domain of f.
- You can use the **horizontal line test** to identify inverse functions. If a horizontal line crosses a function more than once, there is no inverse function.

The graphs of inverse functions

- The graph of the inverse of a function is a reflection of that function in the line $y = x$.
- To find the inverse function algebraically, replace $f(x)$ with y and solve for y.
- The function $I(x) = x$ is called the identity function. It leaves x unchanged. So $f \circ f^{-1} = I$.

Transformations of functions

- $f(x) + k$ translates $f(x)$ vertically a distance of k units upward.
- $f(x) - k$ translates $f(x)$ vertically a distance of k units downward.
- $f(x + k)$ translates $f(x)$ horizontally k units to the left, where $k > 0$.
- $f(x - k)$ translates $f(x)$ horizontally k units to the right, where $k > 0$.
- $-f(x)$ reflects $f(x)$ in the x-axis.
- $f(-x)$ reflects $f(x)$ in the y-axis.
- $f(qx)$ stretches $f(x)$ horizontally with scale factor $\frac{1}{q}$.
- $pf(x)$ stretches $f(x)$ vertically with scale factor p.

Do these exercises on a separate sheet of paper, show all the work, and box in your answer.

Review exercise (Non Calculator section)

1 a If $g(a) = 4a - 5$, find $g(a - 2)$.
  b If $h(x) = \frac{1+x}{1-x}$, find $h(1 - x)$.

2 a Evaluate $f(x - 3)$ when $f(x) = 2x^2 - 3x + 1$.
  b For $f(x) = 2x + 7$ and $g(x) = 1 - x^2$, find the composite function defined by $(f \circ g)(x)$.

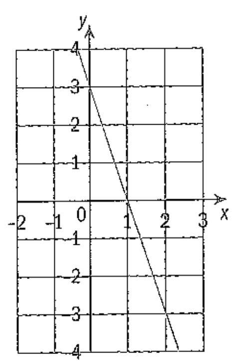
3 Find the inverses of these functions.
  a $f(x) = \frac{3x+17}{2}$
  b $g(x) = 2x^3 + 3$

4 Find the inverse of $f(x) = -\frac{1}{5}x - 1$. Then graph the function and its inverse.

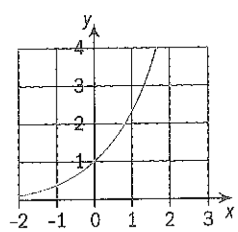
5 Find the inverse functions for
  a $f(x) = 3x + 5$ b $f(x) = \sqrt[3]{x+2}$

6 Copy each graph and draw the inverse of each function.

a

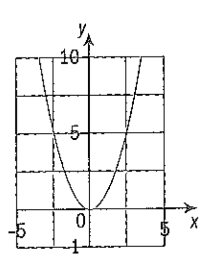


b



7 Find the domain and range for each of these graphs.

a



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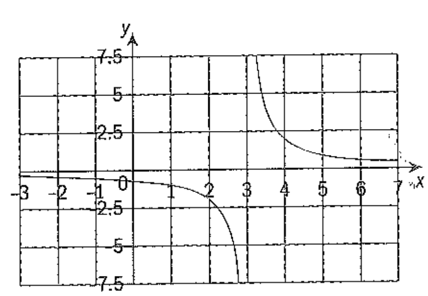


EXAM-STYLE QUESTION

8 For each function, write a single equation to represent the given combination of transformations.

a $f(x) = x$, reflected in the y-axis, stretched vertically by a factor of 2, horizontally by a factor of $\frac{1}{3}$ and translated 3 units left and 2 units up.

b $f(x) = x^2$, reflected in the x-axis, stretched vertically by a factor of $\frac{1}{4}$, horizontally by a factor of 3, translated 5 units right and 1 unit down.

9 **a** Explain how to draw the inverse of a function from its graph.

b Graph the inverse of $f(x) = 2x + 3$.

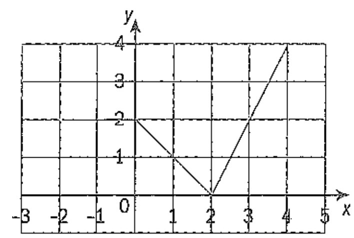
EXAM-STYLE QUESTION

10 Let $f(x) = 2x^3 + 3$ and $g(x) = 3x - 2$.

a Find $g(0)$. **b** Find $(f \circ g)(0)$. **c** Find $f^{-1}(x)$.

EXAM-STYLE QUESTIONS

11 The graph shows the function $f(x)$, for $-2 \le x \le 4$.



a Let $h(x) = f(-x)$. Sketch the graph of $h(x)$.

b Let $g(x) = \frac{1}{2} f(x - 1)$. The point $A(3, 2)$ on the graph of f is transformed to the point P on the graph of g. Find the coordinates of P.

12 The functions f and g are defined as $f(x) = 3x$ and $g(x) = x + 2$.

a Find an expression for $(f \circ g)(x)$.

b Show that $f^{-1}(12) + g^{-1}(12) = 14$.

13 Let $g(x) = 2x - 1$, $h(x) = \frac{3x}{x-2}, x \neq 2$

a Find an expression for $(h \circ g)(x)$. Simplify your answer.

b Solve the equation $(h \circ g)(x) = 0$.

The instruction 'Show that...' means 'Obtain the required result (possibly using information given) without the formality of proof'.
For 'Show that' questions you do not usually need to use a calculator.
A good method is to cover up the right-hand side of the equation and then work out the left-hand side until your answer is the same as the right-hand side.

Review exercise (Calculator section)

1 Use your GDC to sketch the function and state the domain and range of $f(x) = \sqrt{x+2}$.

2 Sketch the function $y = (x+1)(x-3)$ and state its domain and range.

3 Sketch the function $y = \frac{1}{x+2}$ and state its domain and range.

EXAM-STYLE QUESTIONS

4 The function $f(x)$ is defined as $f(x) = 2 + \frac{1}{x+1}$, $x \neq -1$.

a Sketch the curve $f(x)$ for $-3 \leq x \leq 2$.

b Use your GDC to help you write down the value of the x-intercept and the y-intercept.

5 a Sketch the graph of $f(x) = \frac{1}{x^2}$

b For what value of x is $f(x)$ undefined?

c State the domain and range of $f(x)$.

6 Given the function $f(x) = \frac{2x-5}{x+2}$

a write down the equations of the asymptotes

b sketch the function

c write down the coordinates of the intercepts with both axes.

7 Let $f(x) = 2 - x^2$ and $g(x) = x^2 - 2$.

a Sketch both functions on one graph with $-3 \leq x \leq 3$.

b Solve $f(x) = g(x)$.

EXAM-STYLE QUESTIONS

8 Let $f(x) = x^3 - 3$.

a Find the inverse function $f^{-1}(x)$.

b Sketch both $f(x)$ and $f^{-1}(x)$ on the same axes.

c Solve $f(x) = f^{-1}(x)$.

9 $f(x) = e^{2x-1} + \frac{2}{x+1}$, $x \neq 1$.

Sketch the curve of $f(x)$ for $-5 \leq x \leq 2$, including any asymptotes.

10 Consider the functions f and g where $f(x) = 3x - 2$ and $g(x) = x - 3$.

a Find the inverse function, f^{-1}.

b Given that $g^{-1}(x) = x + 3$, find $(g^{-1} \circ f)(x)$.

c Show that $(f^{-1} \circ g)(x) = \frac{x-1}{3}$.

d Solve $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$

Let $h(x) = \frac{f(x)}{g(x)}$, $x \neq 2$.

d **Sketch** the graph of h for $-6 \leq x \leq 10$ and $-4 \leq y \leq 10$, including any asymptotes.

e Write down the **equations** of the asymptotes.

When IB exams have words in **bold** script, it means that you must do exactly what is required. For example the equation could be given as $x = 3$ but not just as 3.