

# IB Math HL, Year 1: Summer Packet

Due on the first day of class

Name: \_\_\_\_\_

## Welcome to IB Math: Analysis and Approaches, higher level!

You are about to begin a rigorous, two-year course that covers topics in Algebra, Functions, Trigonometry, Geometry, Probability, Statistics, and Calculus. The first year of this course plans to cover the basics of each of those topics, so that we can dive deeper in to each topic in Year 2. In Analysis and Approaches (AA), the Year 1 course is taught similarly to an Precalculus course, with some focus on differential calculus toward the end of the year. Next year, you will take the IB Math AA HL exam, a two-year, cumulative exam. If this sounds overwhelming, stop, and take a breath—my job as your teacher is to develop your quantitative reasoning skills and give you the foundation you'll need to succeed in Year 2.

Since so much of this course relies on the concepts you have learned thus far in mathematics, this packet has been designed to help you brush up on your skills so you can hit the ground running in August. This course is rigorous and demanding, and so it is crucial that you enter the course in August with a solid foundation in these topics. You should consider working as independently as possible. However, you are also encouraged to use any online resource at your disposal to help relearn or review these skills (I especially recommend Khan Academy). If you choose to use an online resource, please indicate on which problems you used it, so I know where you had some difficulty. It's okay to get help— in fact, I expect you to in some cases— just be honest about when and where you did.

As of right now, I'm supposed to teach this class in the fall, and **I plan to count this assignment as a series of homework grades. There will also be a quiz on this content within the first few days of class.** I'm using this packet both for you to practice your math skills and for me to figure out any problem areas I should address right away. We will not have extensive time to backtrack on this content, but I will still use this as an important tool in structuring Year 1 of this course.

You must show all work to receive full credit, and work should be completed neatly and thoroughly, preferably in pencil. In the interest of saving some paper, I didn't provide a lot of room to complete these problems, so **please work on separate sheets of paper, and attach them to this packet before submitting.**

If you have any questions at all over the summer, please reach out to me! My email is **rcox@theproutschool.org**. I would be happy to Zoom with you on a case-by-case basis if you're having difficulty.

Have a great summer!

- Mr. Cox

## A brief introduction to IB notations and commands

### Notation

Number sets	$\mathbb{N}$	The set of positive integers and zero (natural numbers), $\{0, 1, 2, 3, \dots\}$
	$\mathbb{Z}$	The set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
	$\mathbb{Z}^+$	The set of positive integers, $\{1, 2, 3, \dots\}$
	$\mathbb{Q}$	The set of rational numbers, any number that can be written as a fraction in simplest form
	$\mathbb{Q}^+$	The set of positive rational numbers, $\{x x \in \mathbb{Q}, x > 0\}$
	$\mathbb{R}$	The set of real numbers
	$\mathbb{R}^+$	The set of positive real numbers, $\{x x \in \mathbb{R}, x > 0\}$
Absolute value	$ x $	IB may refer to this as <i>modulus</i>
Line segments		Line segments $\overline{AB}$ may be written as $[AB]$
Angles		We typically write angle $A$ as $\angle A$ . IB will use the notation $\hat{A}$ or $\hat{C}\hat{A}\hat{B}$
Repeating decimals		Standard notations: $0.\overline{3} = 0.3333\dots$ , $0.\overline{123} = 0.123123\dots$ IB notation: $0.\dot{3}$ , $0.\dot{1}2\dot{3}$
Slope		IB will refer to this as the <i>gradient</i>
Graphing calculator		IB will refer to this as a <i>GDC</i> (graphic display calculator). The TI-83 Plus/TI-84 Plus, as well as similar Casio models, are recommended. The TI-Nspire is prohibited for IB because of the computer algebra system (CAS) installed.

### Key Command Terms

Draw	Represent by means of a labeled, accurate diagram or graph, using a pencil. A ruler should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted and joined in a straight line or curve.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.
Show that	Obtain the required result (possible using the information given) without the formality of proof. These questions do not generally require the use of a calculator.
Sketch	Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Write down	Obtain the answer(s), usually by extracting information. Little to no calculation is required. Working does not need to be shown.

# 1 Functions

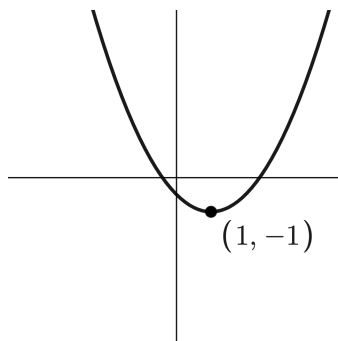
## 1.1 Is it a function?

1-6. Determine if each relation below is a function. If so, state the domain and range in set notation.

1.

$x$	$y$
1	4
2	5
3	5
4	3
5	2

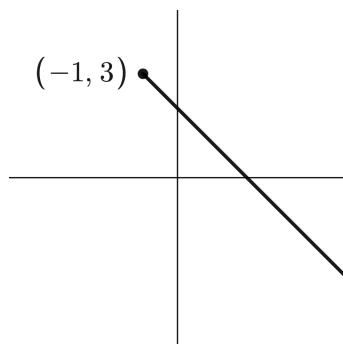
2.



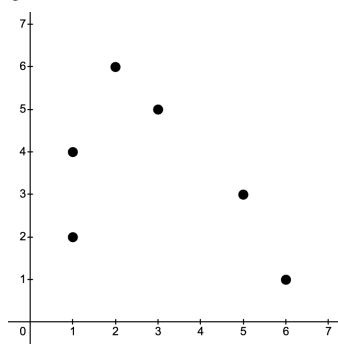
3.

$$\{(0, -1), (2, 1), (1, 2), (3, -1), (0, 2)\}$$

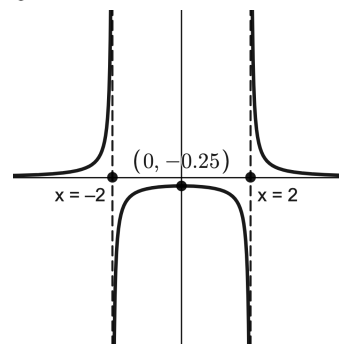
4.



5.



6.



7-9. State the domain and range of each function without the use of a calculator. Express your answers using interval notation. If there is not a point clearly marking the end of the curve, assume that the curve continues infinitely in that direction.

7.  $f(x) = \sqrt{x - 4}$

8.  $g(x) = 5x - 3x^2$

9.  $y = \frac{x + 4}{x - 2}$

## 1.2 Quadratic polynomials, equations, and functions

10-15. Factor each expression completely.

10.  $x^2 - x - 2$

11.  $x^2 + 3x - 4$

12.  $16x^2 - 81y^2$

13.  $3x^2 - 5x + 2$

14.  $2x^2 - x - 6$

15.  $x^3 - 3x^2 - 18x$

**16-21.** Solve each equation by using any method (factoring, completing the square, or the quadratic formula). You may not solve by graphing or by guess-and-check. Give exact answers as solutions.

16.  $x^2 + 25 = 10x$

17.  $x^2 + 3x - 1 = 0$

18.  $x + \frac{12}{x} = 7$

19.  $x^2 + 2 = 9$

20.  $x^2 - 5x = 0$

21.  $36x^2 - 35 = 0$

**22-24.** For each of the following, state the axis of symmetry, vertex, concavity,  $x$ -intercept(s), and  $y$ -intercept(s). Then, sketch the graph of each function, clearly labeling the intercepts and vertex.

22.  $y = -2(x + 2)(x - 1)$

23.  $y = \frac{1}{2}(x - 2)^2 - 4$

24.  $y = 2x^2 + 6x - 3$

**25-29.** Use your graphing calculator<sup>1</sup> for the following questions. Round all approximations to three significant figures.

16. Find the roots of  $3x^2 - x - 5 = 0$ .

17. Find the minimum value of  $f(x) = 2x^2 - 5x + 1$ .

18. Find the maximum value of  $g(x) = -3x^2 + x - 3$ .

19. Find the points of intersection of  $y_1 = 3 - 5x - x^2$  and  $y_2 = x^2 + 3x + 11$ .

20. Find the points of intersection of  $y_1 = x^2 + 3x - 1$  and  $y_2 = 5 - x$ .

### 1.3 Operations between functions

**30-35.** Let  $f(x) = 2x^2 - 1$ ,  $g(x) = 3x$ ,  $h(x) = 5 - x$ . Find each of the following.

30.  $(f + g)(2)$

31.  $(f - g)(x)$

32.  $(hf)(x)$

33.  $(f \circ h)(x)$

34.  $(g \circ h)(4)$

35.  $(f \circ f)(-1)$ .

**36-39.** Find the inverse of each of the following.

36.  $f(x) = 2x + 1$

37.  $f(x) = \frac{x^3}{3}$

38.  $g(x) = \frac{5}{x - 2}$

39.  $g(x) = 1 + \sqrt{4 - x}$

40. If the point  $(2, 7)$  is on the graph of  $f(x)$ , what point must lie on the graph of  $f^{-1}(x)$ ?

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<sup>1</sup>IB will refer to your graphing calculator as a *graphic display calculator*, or GDC.

## 2 Number and algebra

### 2.1 Solving equations

**41-45.** Solve for  $x$ . Eliminate any extraneous solutions, if necessary.

$$41. \sqrt{37 - 3x} = x - 3$$

$$42. -3(2x + 1)^3 = -192$$

$$43. \frac{x}{3} - \frac{5}{2} = \frac{-3}{x}$$

$$44. \frac{4x - 1}{x + 1} = x - 1$$

$$45. 2|3x - 1| + 5 = -2x + 8$$

### 2.2 Solving inequalities

**46-49.** Solve each inequality. Express your answers in interval notation.

$$46. 5(x - 3) \leq 8(x + 5)$$

$$47. 3(6x - 1) > 18 - 3x$$

$$48. 26 + m \geq 5(-6 + 3m)$$

$$49. -2(1 - 5x) > -(x + 1) - 1$$

### 2.3 Solving systems of equations

**50-53.** Solve algebraically (without a graphing calculator).

$$50. \begin{cases} 3x + 7y = 36 \\ x = 5y - 10 \end{cases}$$

$$51. \begin{cases} 6x + 10y = 32 \\ 4x - 2y = 4 \end{cases}$$

$$52. \begin{cases} x = y^2 \\ x - y = 6 \end{cases}$$

$$53. \begin{cases} x^2 + y^2 = 25 \\ y = x^2 - 13 \end{cases}$$

54. On a graph, where does the solution(s) to a system of equations lie?

### 2.4 Simplifying expressions

**55-60.** Simplify each radical expression, rationalizing the denominator where necessary. Provide exact answers.

$$55. \sqrt{72}$$

$$56. \frac{3}{\sqrt{5}}$$

$$57. \frac{1}{\sqrt{x}}$$

$$58. \frac{1}{1 + \sqrt{3}}$$

$$59. \frac{1}{\sqrt{x} - 2}$$

$$60. \sqrt{48x^6}$$

60-62. Simplify without the use of a calculator.

60.  $8^{2/3}$

61.  $81^{-3/4}$

62.  $(9x^2)^{1/2}$

63-65. Simplify completely, leaving only positive exponents in your answer.

63.  $(5x^2y)(2x^4y^{-3})$

64.  $\left(\frac{4x^5y}{16xy^4}\right)^3$

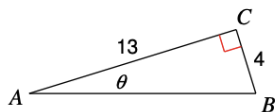
65.  $\frac{2x^4y^{-4}}{8x^7y^3}$

### 3 Geometry and Trigonometry

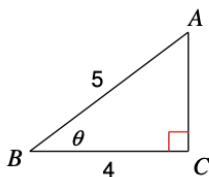
#### 3.1 Right triangle trigonometry

66-71. Use the Pythagorean Theorem to find the missing side of the right triangle. Then, find the exact value of the indicated trigonometric ratio.

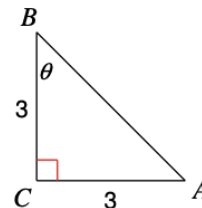
66.  $\sin \theta$



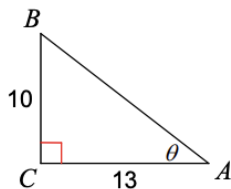
67.  $\cos \theta$



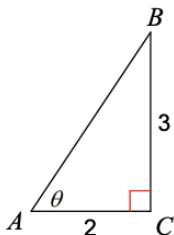
68.  $\tan \theta$



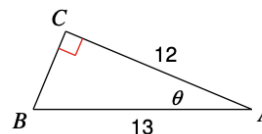
69.  $\sin \theta$



70.  $\cos \theta$

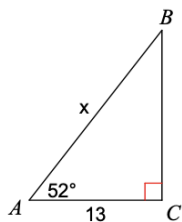


71.  $\tan \theta$

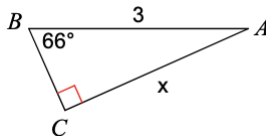


72-73. Use a trigonometric ratio to find the measure of the indicated side. Round your answers to three significant figures.

72.



73.



## 4 Something new: rational expressions

### 4.1 Warming up

Rational expressions are just *algebraic fractions*, so before we begin, work with these fractions. Think especially about the *process* you follow.

**74-77. Simplify, reducing your answers completely.**

$$74. \frac{2}{3} + \frac{1}{5}$$

$$75. \frac{2}{3} \cdot \frac{6}{5}$$

$$76. \frac{7}{3} \div \frac{5}{4}$$

$$77. \frac{11}{15} - \frac{9}{10}$$

We have been dealing with rational expressions since elementary school...we've just been calling them fractions until now! The term "rational expression" is a more general term, since "fraction" often refers to expressions with numbers (e.g.  $\frac{1}{2}$ ,  $\frac{7}{3}$ ). When we add, subtract, multiply, or divide rational expressions, we work with them as if they were just more complicated fractions.

### 4.2 Addition and subtraction

When adding or subtracting rational expressions, we must find a common denominator first, before adding across. Like with fractions, terms must have like denominators before we add or subtract.

$$\begin{aligned} \frac{2}{x+1} + \frac{3}{x} &= \frac{2}{x+1} \cdot \frac{x}{x} + \frac{3}{x} \cdot \frac{x+1}{x+1} && \text{Make a common denominator} \\ &= \frac{2x}{x(x+1)} + \frac{3(x+1)}{x(x+1)} \\ &= \frac{2x + 3x + 3}{x(x+1)} && \text{Distribute and add} \\ &= \frac{5x + 3}{x^2 + x} && \text{Simplify} \end{aligned}$$

**78-81. Add or subtract. All answers should be reduced fully. In some problems, it may be easier to factor and simplify *before* completing the problem.**

$$78. \frac{3}{1-x} + \frac{5}{1+x}$$

$$79. \frac{x}{x+5} - \frac{2}{x-3}$$

$$80. \frac{2x}{x^2-9} + \frac{4}{x+3}$$

$$81. \frac{-1}{x} + \frac{2}{x^2+1} + \frac{x+1}{x^3+x}$$

### 4.3 Multiplication and division

Recall that division of rational expressions is multiplication by the divisor's reciprocal. You may have heard the expression “keep, change, flip” from past teachers. With rational expressions, however, we may see it written as a *complex fraction*. The most helpful thing to do with a complex fraction is rewrite as “normal” division.

$$\begin{aligned}\frac{\frac{x}{x+5}}{\frac{3}{x-2}} &= \frac{x}{x+5} \div \frac{3}{x-2} && \text{Rewrite as division} \\ &= \frac{x}{x+5} \cdot \frac{x-2}{3} && \text{Reciprocal of the second term, multiply} \\ &= \frac{x \cdot (x-2)}{(x+5) \cdot 3} \\ &= \frac{x^2 - 2x}{3x + 15} && \text{Simplify}\end{aligned}$$

**82-85.** Multiply or divide. All answers should be reduced fully. In some problems, it may be easier to factor and simplify *before* completing the problem.

$$\begin{array}{ll}82. \frac{x^2 + 2x - 3}{x + 2} \cdot \frac{x^2 + 2x}{x^2 - 1} & 83. \frac{\frac{(x+2)^2}{6x^2}}{\frac{x^2 - 4}{3x}} \\ 84. \frac{\frac{x^2 - 14x + 49}{x^2 - 49}}{\frac{3x - 21}{x + 7}} & 85. \frac{x^2 + xy - 2y^2}{x^3 + x^2y} \cdot \frac{x}{x^2 + 3xy + 2y^2}\end{array}$$