IB Math AA HL Year 2 Summer Packet #1

Part 1: Venn Diagrams

3 Video Links:

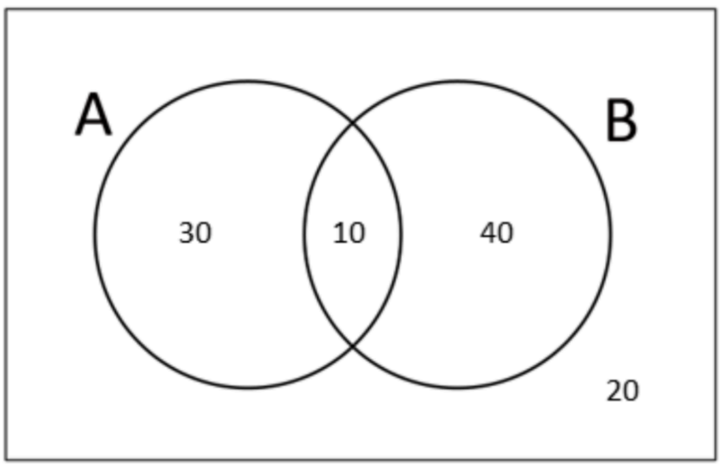
1. Shading Venn Diagrams 2
2. Venn diagram problems
3. Venn diagrams and probability

Complete:

Problem solving with Venn diagrams

John Venn originally used his diagrams to solve problems. He did this by listing the number, rather than the elements, in each set.

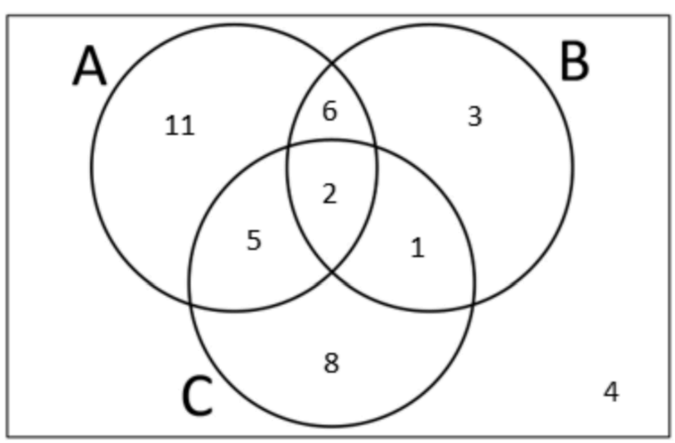
Below is a simple two way Venn diagram.
Set A is the people who do athletics in a school and Set B is the people who do basketball.



What can we tell from the Venn diagram?

Problem solving with Venn diagrams

The Venn diagram below shows the choice made by students at a school to do either, Arabic (A), Bahasa Malaysian (B) or Cantonese (C) as a language option.



a) How many students are there at the school?

b) How many students made no language choice?

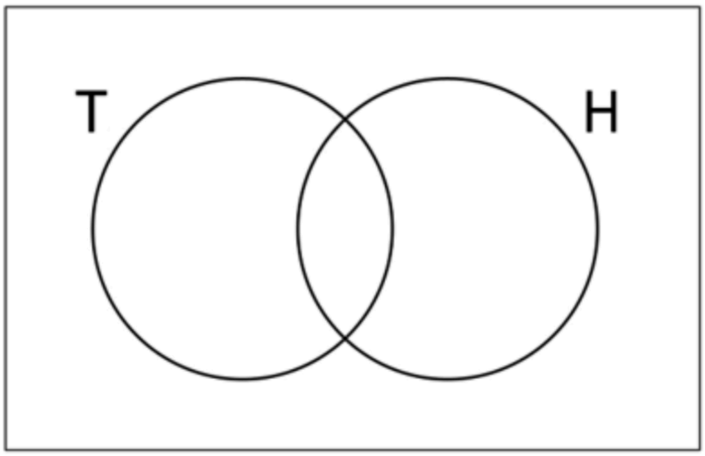
c) How many students chose Arabic and Cantonese, but not Bahasa?

Problem solving with Venn diagrams

In a school 100 students get to choose between tennis (T) or hockey (H). The students could choose to do both or they could choose neither.

15 students choose neither sport, 45 students choose tennis and 60 students choose hockey.

Complete the Venn diagram below to show the number of students who take each sport.



Questions

1. U is the set of all positive integers from 20 to 30 inclusive.

A={prime numbers} B={odd numbers} C={multiples of 3}

a) Write down the number of elements in $A \cap B$.

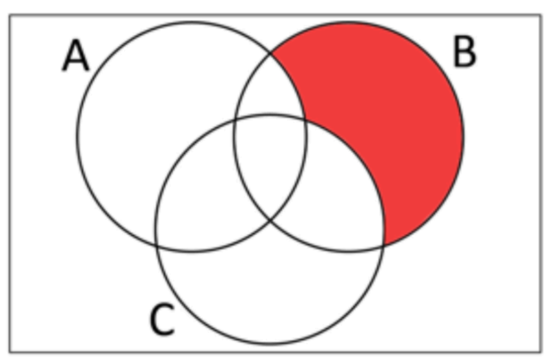
b) Write down the number of elements in $B \cap C$.

c) What set notation symbol could you use to describe $A \cap B$?

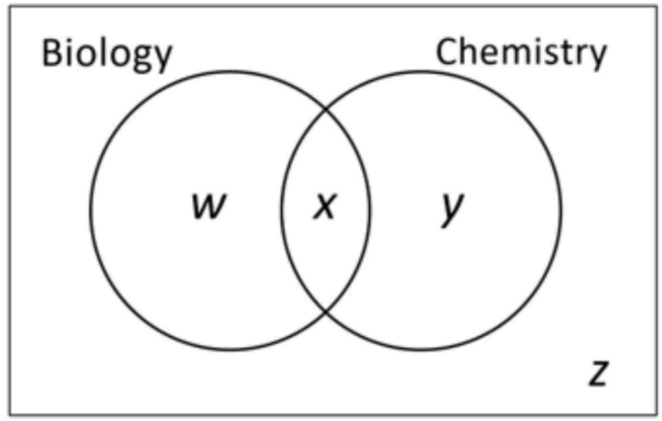
d) List the element of $A \cup B$.

e) List any element of $(A \cup B \cup C)'$.

f) List the elements that would be in the shaded part of the Venn diagram below.



2. In a group of 24 students, 16 take Biology and 9 take Chemistry. Four students take neither Biology or Chemistry. The Venn diagram below shows the choices. The values of *w, x, y* and *z* represent numbers of students.



a) Find the values of *w, x, y* and *z*.

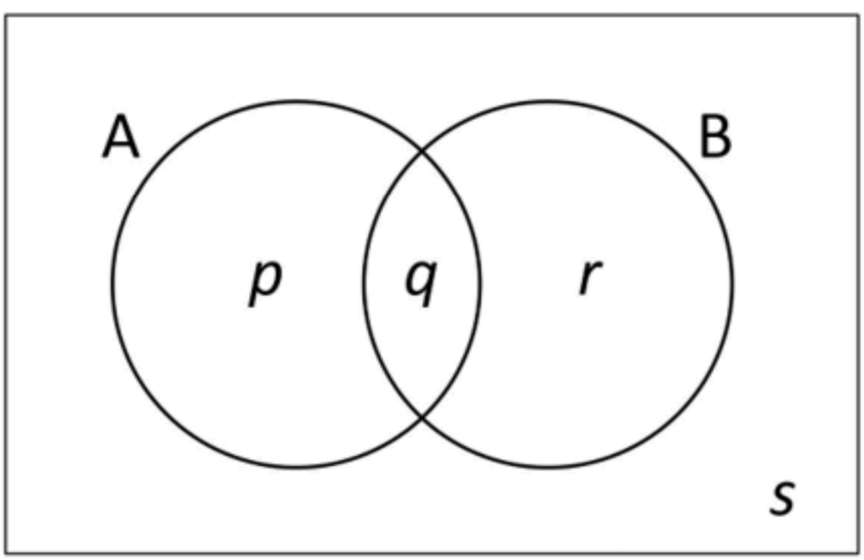
b) How many students take both Biology and Chemistry?.

c) How many students take only Chemistry?

d) How many students take only one of the two subjects?

3. The Venn diagram below shows events A and B from a Universal set of 30.

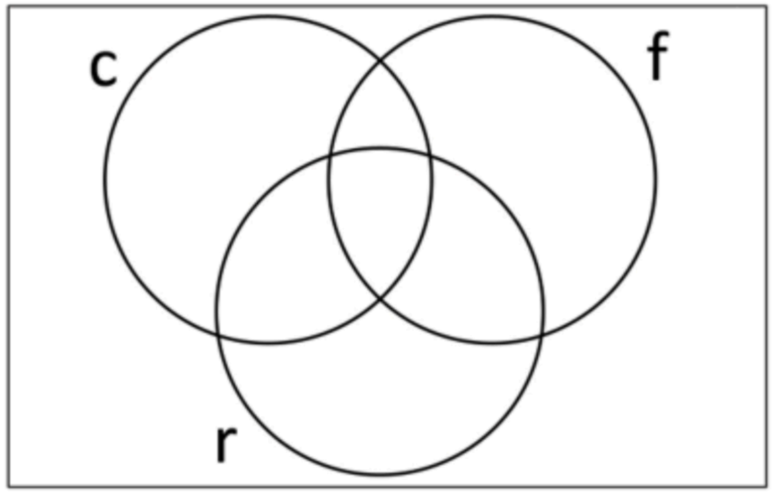
Given that $n(A \cup B)' = 5$, $n(A)=20$, $n(B)=12$, find the values of p, q, r, and s.



4. In a school boys can do cricket, football or rugby, or any combination or none of the sports.

Their choices are shown below.

5 do cricket only,
15 do football only,
12 do rugby only,
6 do both cricket and football, but not rugby,
18 do both cricket and rugby but not football,
33 do cricket,
7 do none of these sports,
there are 70 boys at the school.



Copy the Venn diagram and complete with information to show the number of boys who choose each sport.

Part 2: Independence

4 Video Links:

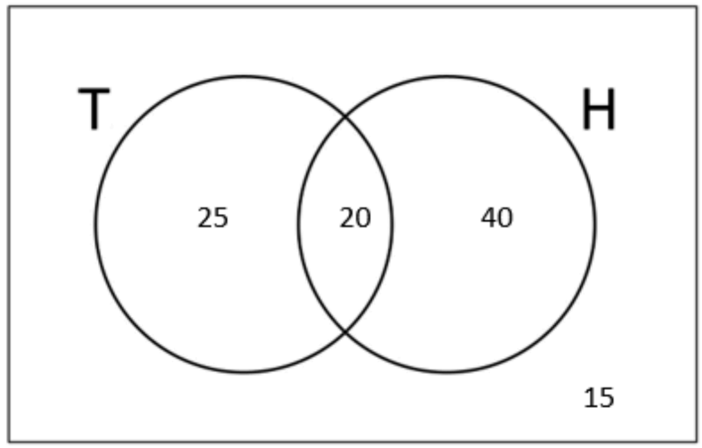
1. Mutually Exclusive
2. Independent events
3. Venn Probability
4. Venn Diagrams

Addition Law

In a school 100 students get to choose between tennis (T) or hockey (H). The students could choose to do both or they could choose neither.

15 students choose neither sport, 45 students choose tennis and 60 students choose hockey.

This information is shown in the Venn diagram.



How many students do tennis or hockey?

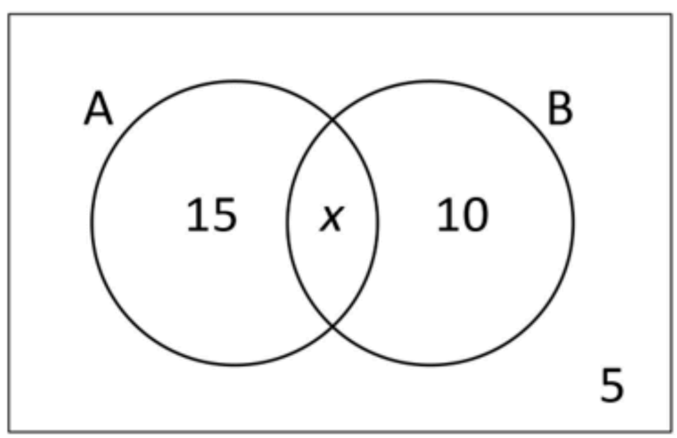
This is actually the notation $(T \cup B)$.

Mutually Exclusive

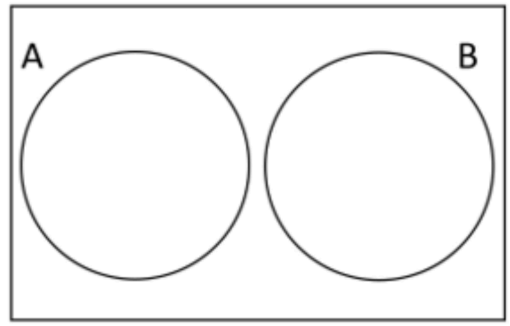
In a school 30 students get to choose between Arabic and or Bengali as a language. The languages are timetabled at the same time on the school timetable.

15 students choose Arabic, 10 do Bengali and 5 do neither.

This information is shown in the Venn diagram opposite.



If events A and B are **mutually exclusive** if being in one group prevents you from being in the other group.



Questions

1. $P(A) = 0.5$, $P(A \cup B) = 0.85$ and $P(B') = 0.35$. By use of a Venn diagram determine if the sets are mutually exclusive.

2. $P(A) = 0.6$, $P(A \cup B) = 0.9$ and $P(B') = 0.7$. By use of a Venn diagram determine if the sets are mutually exclusive.

Independence

_______________________________occurs in probability when the occurrence of one event makes it neither more nor less probable that the other occurs.

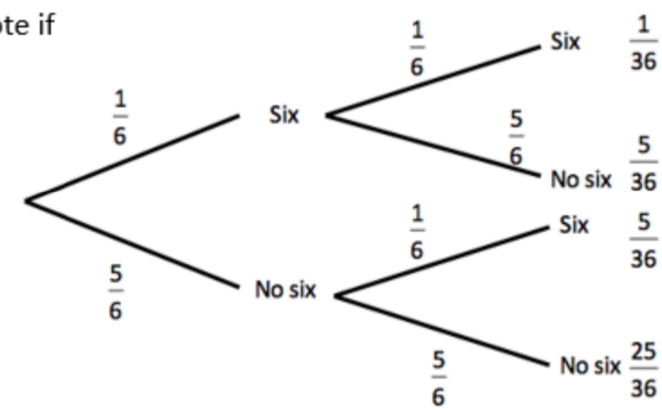
For example when an unbiased dice is rolled twice the outcome of the second roll is in no way affected by the outcome of the first roll – ________________________________.

However, if a card is randomly chosen from a pack of normal playing cards and then not replaced, and a second card is then randomly chosen ________________________________. The first card has some effect on the outcome of the second card.

If two events are independent then the following is true.

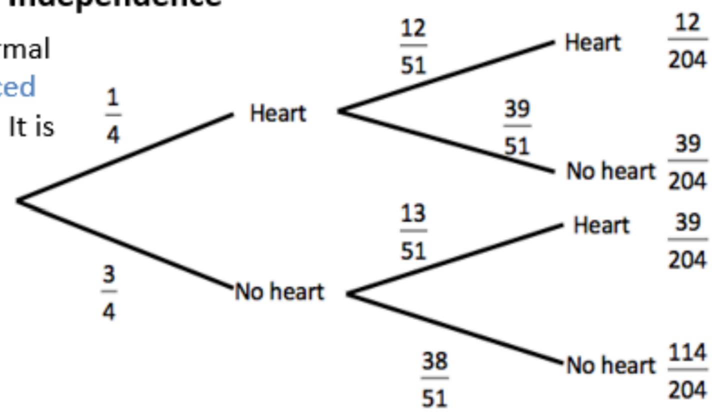
Independence

An unbiased die is rolled twice and we note if a six occurs. The tree diagram opposite shows the possibilities and their probabilities.
Are the events independent?



Independence

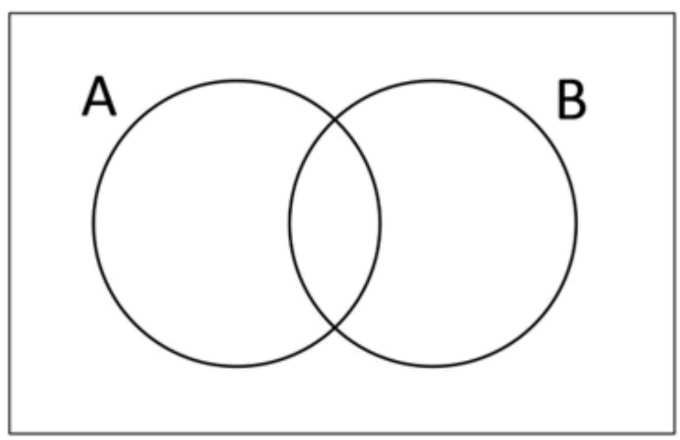
An card is randomly chosen from a normal pack of 52 cards. The card is not replaced and a second card is randomly chosen. It is noted if the cards are hearts or not.
Are the events independent?



Independence – an example question

Events A and B are independent.
$P(A) = 0.2$ and $P(A \cup B)' = 0.3$.
Find the value of x if $P(B) = x$.

Start by drawing a Venn diagram.



Questions

1. Events A and B are mutually exclusive.

$P(A)=0.3$ and $P(B)=0.6$.

Find,

a) $P(A\cap B)$

b) $P(A\cup B)$

c) $P(A\cap B')$

d) $P(A'\cup B)$.

2. $P(A)=\frac{2}{5}$, $P(B)=\frac{1}{2}$, and $P(A\cup B)=\frac{7}{10}$.

Determine if events A and B are independent.

3. $P(A\cap B)=0.36$, $P(A'\cap B)=0.24$, and $P(A\cup B)'=0.34$.

a) Draw a Venn diagram to illustrate the above probabilities.

b) Write down $P(A)$ and $P(B)$.

c) Determine if events A and B are independent.

Part 3: Tree Diagrams

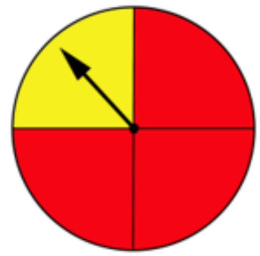
3 Video Links:

1. Tree diagrams 1
2. Tree diagrams 2
3. [Tree diagrams 3]

Constructing a tree diagram

The spinner opposite is spun twice and the colour noted.

We can show all the possibilities by constructing a tree diagram to represent our sample space.



Some points to note about tree diagrams:

-
-
-
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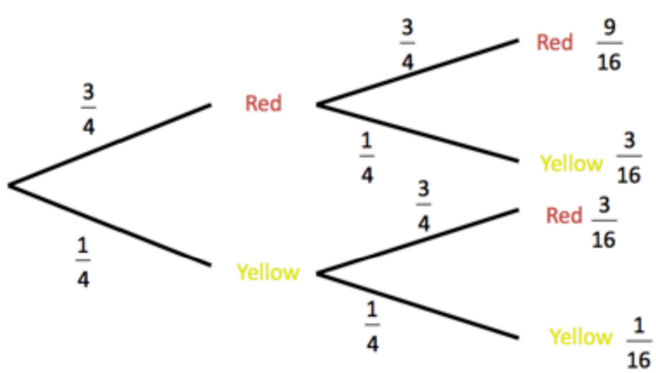
Constructing a tree diagram

Use the tree diagram to calculate the probability of the spinner landing on,

a) red twice,

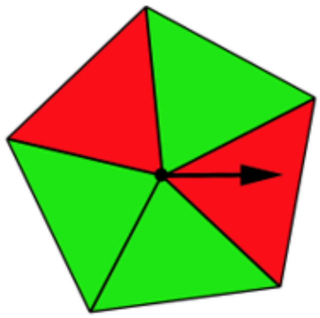


b) two colours the same,

c) a red and a yellow.

Constructing a tree diagram

The spinner opposite has 5 equal areas and when the spinner is spun it has an equal probability of landing in any section.

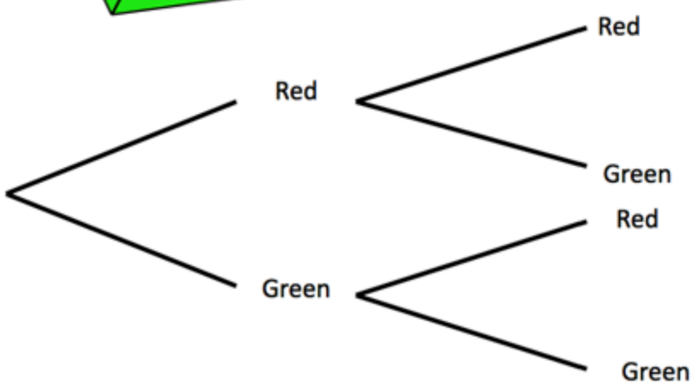


a) Copy and complete the tree diagram above to show all the possibilities of two spins on the spinner.

Use the tree diagram to find the probability of obtaining,

b) two greens,

c) two different colours.



Conditional tree diagrams

Outcomes of events can be dependent on previous events.

The probability of rain on any day in Manchester is 0.7. If it rains the probability of a road accident is 0.6, if there is no rain the probability of a road accident is 0.2.

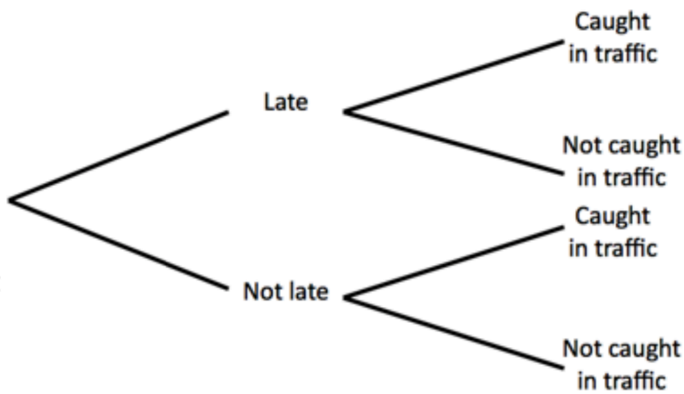
Draw a tree diagram to illustrate the data given.

a) Use the tree diagram to find the probability of there being no accident in Manchester on any given day.

b) Use the tree diagram to find the probability of there being an accident in Manchester on any given day.

Conditional tree diagrams

A teacher wakes up late with a probability of $\frac{2}{5}$. If he is late the probability that he will get caught in traffic is $\frac{5}{6}$. If he is not late the probability that he will get caught in traffic is $\frac{1}{3}$.



a) Complete the tree diagram above.

Use the tree diagram to find the probability that,

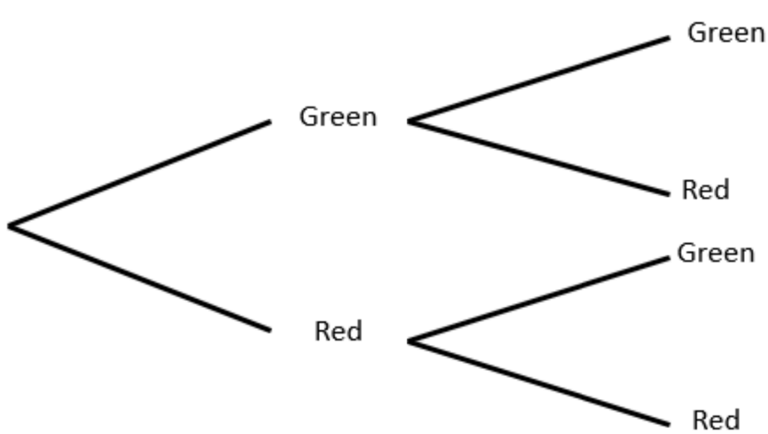
b) the teacher will be caught in traffic,

c) not late and caught in traffic.

Replacement

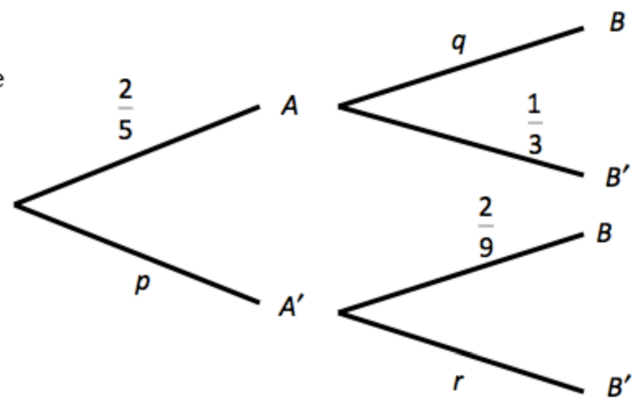
A bag contains 4 green balls and 2 red balls. A ball is picked at random and then replaced. A second ball is picked at random.

Complete the tree diagram below to show all the possibilities.



Questions

1. The diagram below shows the probabilities for events *A* and *B*.



a) Find the values of *p, q*, and *r*.

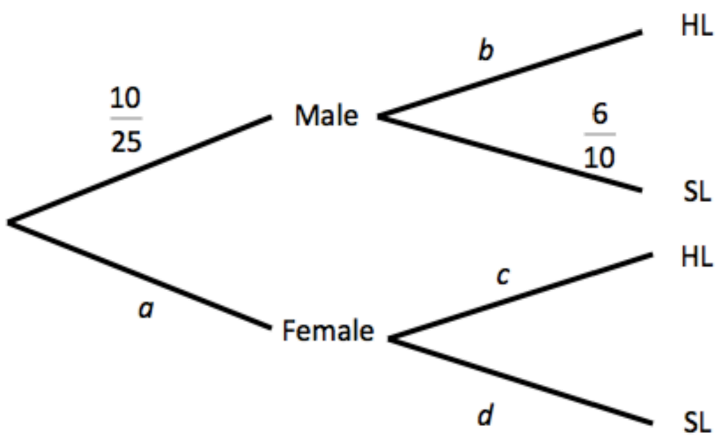
b) Calculate the probability both events taking place.

c) Calculate the probability of one event only taking place.

2. The table below shows 25 students in an IB year group. A student is picked at random and it is noted if they are male or female. Next the student is asked if they do HL or SL Maths. A tree diagram is drawn to show all the possibilities.

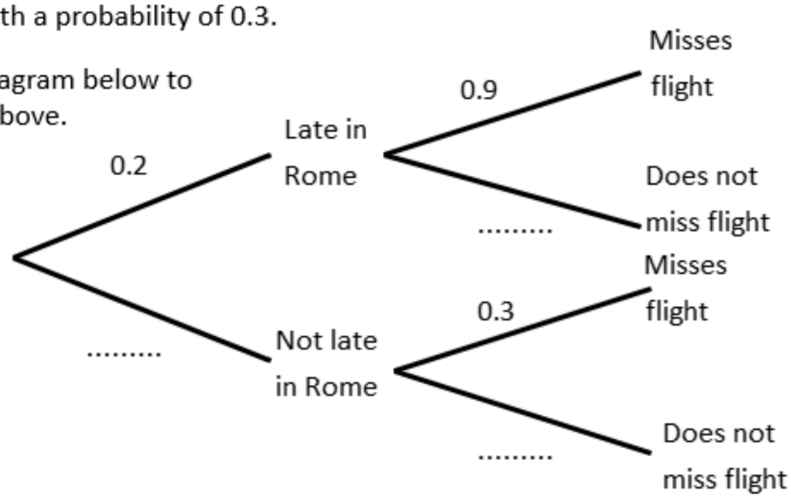
	HL	SL	Total
Male	4	6	10
Female	10	5	15
	14	11	25

Find the values of *a*, *b*, *c* and *d*.



3. Frank is flying from London to Athens, but must change planes in Rome. The probability of the flight arriving late in Rome from London is 0.2. If the flight is late the probability rank missing the connection to Athens of 0.9. If the flight is not late Frank may still miss the connection to Athens with a probability of 0.3.

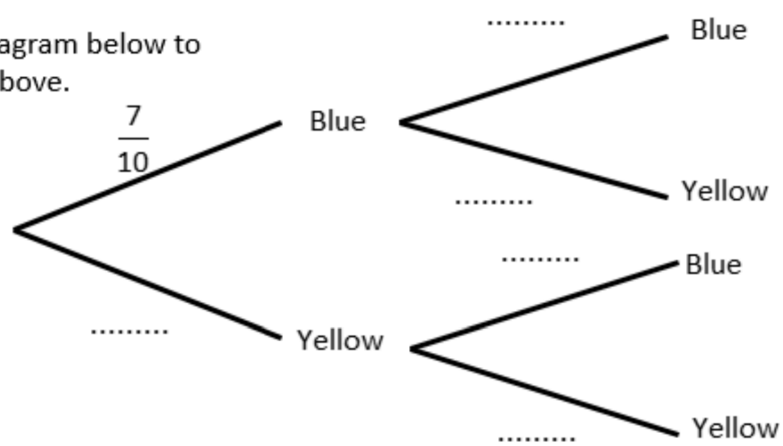
a) Complete the tree diagram below to show the information above.



b) Calculate the probability of Frank missing his flight to Athens.

4. 10 balls are placed in a counters. 7 are blue and 3 are yellow. Two counters are taken at the same time.

a) Complete the tree diagram below to show the information above.



b) Calculate the probability of picking counters of different colours.

c) Calculate the probability of picking counters of the same colour.

Conditional probability

Conditional probability occurs when we are given additional information about data.

Often it allows us to reduce the sample space from the extra information.

Conditional probability is noted by the use of given that in questions, and it can be used with tables, Venn diagrams and tree diagrams.

Conditional probability from tables

The table opposite shows 25 students in an IB year group.

	HL	SL	Total
Male	4	6	10
Female	10	5	15
	14	11	25

How many students are there?

How many students do SL?

How many female students are there?

The table opposite shows 25 students in an IB year group.

	HL	SL	Total
Male	4	6	10
Female	10	5	15
	14	11	25

A student is picked from the group. Given that the student is female, find the probability that the student does HL maths.

Although we still picked a student from a group of 25, we are told some extra information from the given that.

We now know that the student is female so the denominator of the probability is now 15, as there are 15 females.

$$\frac{}{15}$$

There are 10 female students who do HL,
So the probability will be,

$$\frac{10}{15}$$

The table opposite shows 25 students in an IB year group.

	HL	SL	Total
Male	4	6	10
Female	10	5	15
	14	11	25

A student is picked from the group. Given that the student is female, find the probability that the student does HL maths.

Another method is to find the probability of picking a female student first (the given that part). This becomes the denominator.

The numerator is the probability of a picking a female who does HL.

Conditional probability formula

There is a simple formula for conditional probability.

$$P(A/B)=\frac{P(A\cap B)}{P(B)}$$

$P(A/B)$ The probability of A given B.

$P(A\cap B)$ The probability of A and B - the intersection.

$P(B)$ The probability of B.

The table opposite shows 25 students in an IB year group.

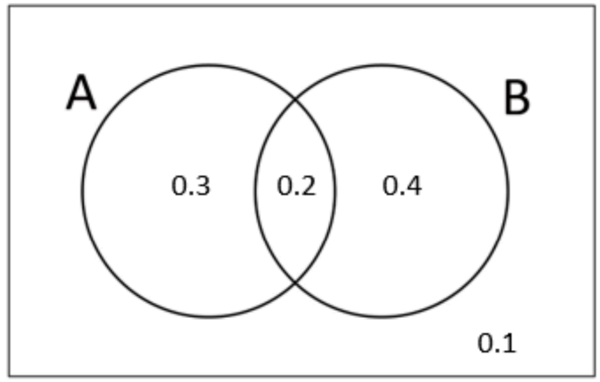
A student is picked from the group. Given that the student is female, find the probability that the student does HL maths.

	HL	SL	Total
Male	4	6	10
Female	10	5	15
	14	11	25

$P(\text{takes HL / female})$ student takes HL given that female.

Conditional probability and Venn diagrams

The Venn diagram opposite shows the probabilities for events A and B.



a) Find P(A).

b) Find P(B).

c) Find $P(A \cap B)$.

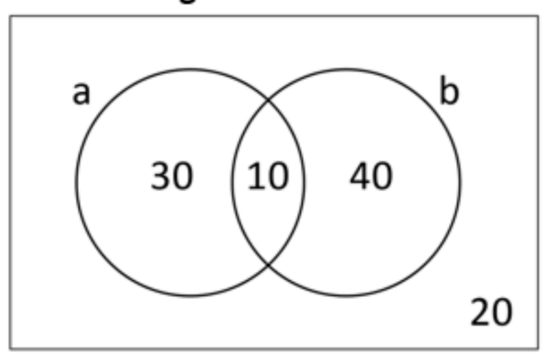
d) Find $P(A / B)$.

e) Find $P(B / A)$.

Conditional probability and Venn diagrams

The Venn diagram below show the choices made by 100 students in a school. The students could choose athletics (a) or basketball (b). They could also opt to take both sports or neither.

Calculate the probability that,



a) the student does athletics given that the student plays basketball,

b) the student plays basketball given that the student does athletics,

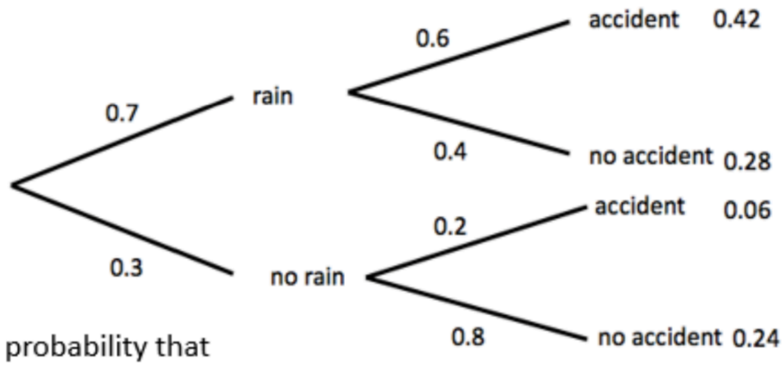
c) the student does both athletics and basketball given that they do at least one sport,

d) the student only does athletics, given that the student does athletics,

e) the student only does basketball, given that the student plays basketball.

Using tree diagrams

The probability of rain on any day in Manchester is 0.7. If it rains the probability of a road accident is 0.6, if there is no rain the probability of a road accident is 0.2.
A tree diagram to show the information is shown below.



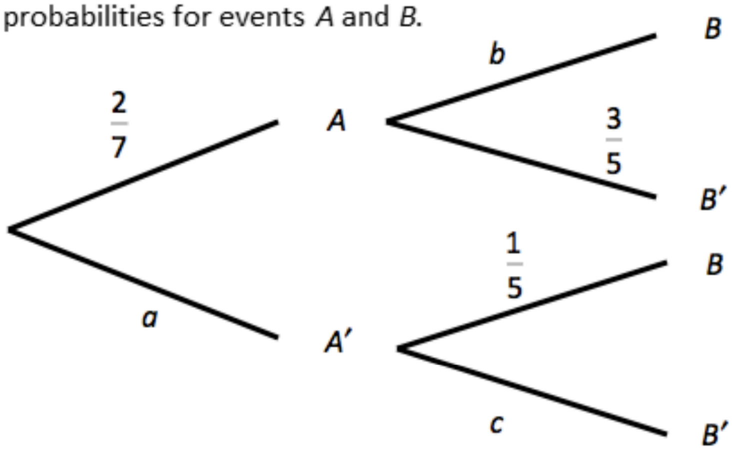
Use your tree diagram to find the probability that it rained given that an accident occurred.

Use your tree diagram to find the probability that it rained given that no accident has occurred.

Questions

1. The diagram below shows the probabilities for events A and B.

   a) Write down the values of a, b, and c.

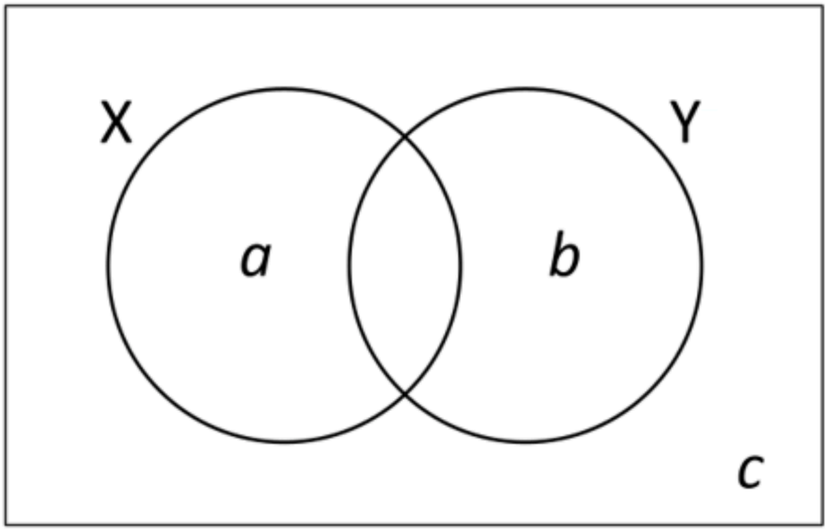


   b) Find $P(B)$.

   c) Find $P(A/B)$.

   d) Find $P(A'/B')$.

2. a) Given that $P(X) = 0.4$, $P(Y) = 0.8$ and $P(X \cap Y) = 0.32$ find the values of a, b and c in the Venn diagram below.



b) Find $P(X / Y)$.

c) Find $P(X / Y')$.

d) Find $P(Y' / X')$.

IB Math HL Year 2 Summer Packet 2024 Name:________________________________

Packet 1 DUE BY JULY 31ST!!

Packet 2 DUE FIRST DAY OF CLASS!!

Dear IB Math HL Student,

Welcome back to Year 2 IB Math HL! The focus this year will be mostly on Calculus. Other topics include Vectors, Probability, Statistics, Exponential and Logarithmic Functions. With the main branch of our content in Calculus we will need to take all the mathematical skills we already have to a deeper level.

To help prepare us for Calculus, there are certain algebraic skills that we will have to have under our belts to ensure understanding of the extensions within the Calculus content. Packet 2 of your summer packet will review some of those foundational skills as well as review some content we learned in Year 1. Please resource all of your Year 1 guided notes to help you through this portion of the summer packet. Packet 1 focuses mostly on new content. It will be presented with guided notes on a slide show along with videos as an introduction. Start each part with the short video clips, then complete the notes with the slide show. At the end of each slide show there are exercise to complete.

Both summer packets must be completed on <u>PRINTED COPIES.</u> SHOW ALL NOTES AND WORK WHERE NECESSARY!

SUBMIT <u>Section 1 BY JULY 31ST</u> ON GOOGLE CLASSROOM AND BRING <u>Section 2 TO THE FIRST DAY OF CLASS</u>!!!

BOTH PARTS OF THE SUMMER PACKET WILL BE GRADED!

AN ASSESSMENT WILL TAKE PLACE WITHIN THE FIRST WEEK OF SCHOOL!!

HAVE A GREAT SUMMER!!!!

Mrs Steere

Packet #2: **Due First Day of School** **Name:___________________________________**

1. Consider the sequence 6, 17, 28, 39, 50,….

a. Find the formula for the nth term:

b. Find the 50th term:

c. Is 325 a term is the sequence?

d. Is 761 a term?

e. Find the sum of the first 20 terms.

2. Consider the sequence 12, -6, 3, -1.5, ….

a. Find the formula for the nth term:

b. Hence, find the 13th term:

c. Find the sum of the first 10 terms.

d. Find the infinite sum of the sequence.

3. Find the sum

$$\sum_{r=1}^{4}(3r-5)$$

4. Find the sum (hint: use a summation formula)

$$\sum_{i=1}^{15}50(.8)^{i-1}$$

5. Simplify without using a calculator. Leave exponents positive.

a. $(-3a^7)^2$

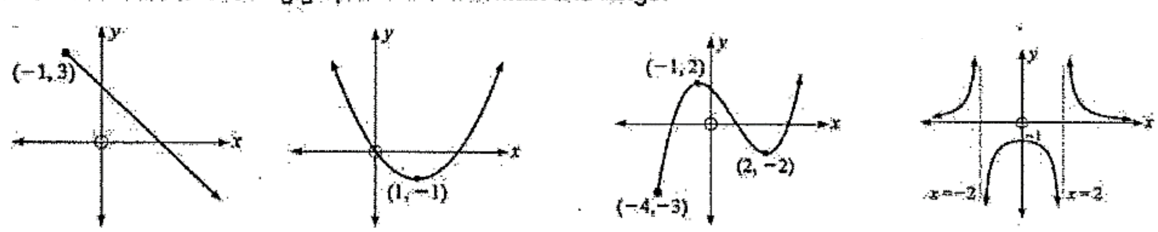
b. $\frac{8ab^5}{2a^ab^4}$

c. $\frac{2^{x+1}}{2^{1-x}}$

d. $27^{\frac{4}{3}}$

e. $(\frac{2a^{-1}}{b^2})^3$

6. For each of the following graphs find the domain and range:



7. For each find the domain without a calculator.

a. $f(x) = \sqrt{4-x}$ b. $f(x) = 5x - 3x^{\frac{1}{2}}$ c. $f(x) = \frac{x^2-4}{x-2}$

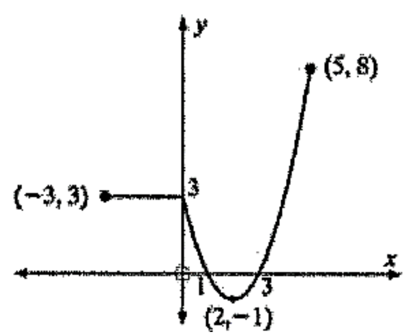
8. Given $f(x) = x^2 + 1$ and $g(x) = 3 - x$ find in the simplest form

a. $(f \circ g)(x) =$ b. $(g \circ f)(x) =$ c. $(f \circ g)(4) =$

9. For the graph of $y = f(x)$, sketch graphs of:

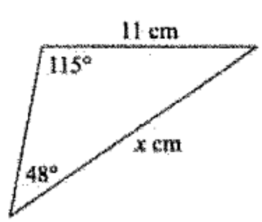
a. $y = f(-x)$ b. $y = -f(x)$

c. $y = f(x+2)$ d. $y = f(x) + 2$

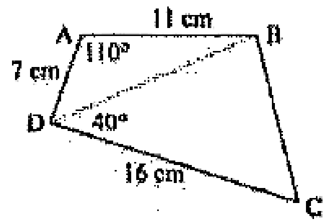


10. Given the point $(3,6)$ on a function $f(x)$. If $g(x) = 2f(x-3) - 5$, find the point $(3,6)$in $g(x)$

11. Find x



12. Find the area of quadrilateral ABCD:



13. A sector has an angle of 68.2° and an area of $20.8\ \text{cm}^2$. Find: (perimeter = arc length + radius + radius)

a its radius b its perimeter.

14. Evaluate *without* a calculator (extremely important concept)

a) Sin 60° b) Cos 45° c) Tan 150° d) Sin 300° e)Cos 210° f)Tan 315°

g) Sin π/6 h) Cos π i) Tan 5π/6 j) Sin 3π/2 k) Cos 5π/4 l) Tan 11π/6

15. Without using a calculator, evaluate:

a) $\sin 30^{\circ} \cos 60^{\circ}$ b) $\cos^2(\frac{\pi}{4}) - \sin(\frac{7\pi}{6})$ c) $1 - \cos^2(\frac{\pi}{6})$

16. Without using a calculator, find:

a) $\cos\theta$ if $\sin\theta = -\frac{1}{\sqrt{2}}$, $\pi < \theta < \frac{3\pi}{2}$. b) $\sin\theta$ if $\cos\theta = \frac{2}{3}$, $0 < \theta < \frac{\pi}{2}$

c) Find $\sin x$ and $\cos x$ given that: $\tan x = -\frac{4}{3}$ and $\frac{\pi}{2} < x < \pi$

17. Solve *without* a calculator: Find θ in radians if $0 \leqslant \theta \leqslant 2\pi$ and:

a) $\cos\theta = \frac{1}{2}$ b) $\sin\theta = 1$ c) $\cos\theta = -1$ d) $\sin 2x = \frac{1}{2}$

18. Use technology to solve for $0 < x < 8$; give answers to 3 significant figures

a) $\sin 2x = 0.162$ b) $\cos(x - 1.3) = -0.609$

19. Solve by factoring. Find x in radians if $0 \leq x \leq 2\pi$

a) $\sin^2 x + \sin x - 2 = 0$ b) $2\sin^2 x + 1 = 3\sin x$

20. If $\sin A = \frac{4}{5}$ and $\cos A = \frac{3}{5}$ find the values of:

a $\sin 2A$ b $\cos 2A$

21. If $\cos\beta = \frac{2}{5}$ where $\frac{3\pi}{2} < \beta < 2\pi$, find the value of $\sin\beta$ and hence the value of $\sin 2\beta$.

22. Without using technology draw the graphs of the following for $0 \leqslant x \leqslant 2\pi$:

i $y = 3\sin x$ $y = \cos 2x$ $y = -\frac{3}{2}\sin x$

$y = \sin x + 2$ $y = \sin(x + \frac{\pi}{4})$

23. (No GDC) Prove by contradiction that if n^3+3 is odd then n is even, $\forall n \in \mathbb{Z}^+$.

24. (No GDC) Use induction to prove that $5^{2n-1}+1$ is divisible by 6, $\forall n \in \mathbb{N}$.

25. Find the term in x^6 in the expansion of $\left(x^2 - \frac{1}{x}\right)^6$.

26. (No GDC) Consider the functions f and g where $f(x) = 3x - 5$ and $g(x) = x - 2$.

(a) Find the inverse function f^{-1}.

(b) Given that $g^{-1}(x) = x + 2$, find $(g^{-1} \circ f)(x)$.

(c) Given also that $(f^{-1} \circ g)(x) = \dfrac{x+3}{x}$, solve $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$.

27. (No GDC) The function f is defined by $f(x) = \dfrac{ax+b}{cx+d}, x \neq -\dfrac{d}{c}$. The function g is defined by $g(x) = \dfrac{2x-3}{x-2}, x \neq -2$.

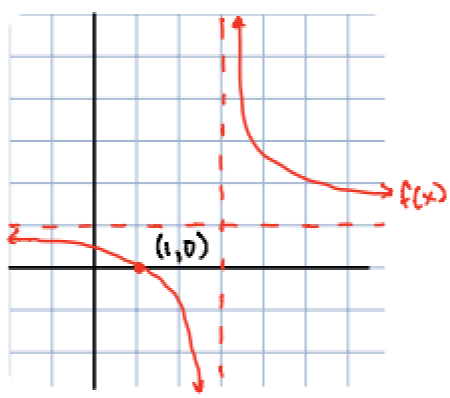
(a) Find the inverse function f^{-1} and the domain of f^{-1}.

(b) Express $g(x)$ in the form $A + \dfrac{B}{x-2}$ where A, B are constants.

(c) Sketch the graph of $y = g(x)$. State the equations of any asymptotes and the coordinates of any intercepts with the axes.

(d) The function h is defined by $h(x) = \sqrt{x}, x \geq 0$. State the domain and range of $h \circ g$.

28. (No GDC) A rational function is defined by $f(x) = a + \dfrac{b}{x-c}$ where the parameters $a, b, c \in \mathbb{Z}$ and $x \in \mathbb{R}, x \neq c$. The graph below is the graph of $y = f(x)$.



(a) Using the information on the graph, state the values of a and c.

(b) Express the translation of the parent function $y = \dfrac{1}{x}$ using the column vector $\begin{pmatrix} h \\ k \end{pmatrix}$.

(c) Find the value of b.

29. (No GDC) The cubic polynomial $3x^3+px^2+qx-2$ has a factor $(x+2)$ and leaves a remainder 4 when divided by $(x+1)$. Find the values of p and q.

30. (No GDC)

(a) Show that the complex number i is a root of the equation $x^4 - 5x^3 + 7x^2 - 5x + 6 = 0$.

(b) Hence or otherwise, find the other roots of this equation.

31. (GDC to check only) Let $z_1 = 2+3i$, $z_2 = 1-4i$, and $z_3 = 2+i$. Find $\dfrac{z_1+z_2}{z_3^*}$.

32. (No GDC) Verify the following identities.

(a) $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = 2\sec^2\theta$

(b) $\dfrac{1}{\tan\theta + \cot\theta} = \sin\theta\cos\theta$